

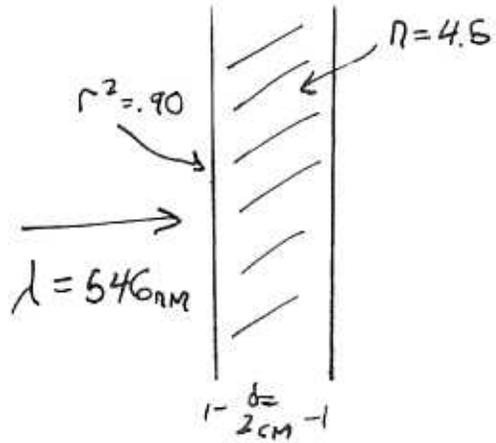
Physics 110B Homework #10

#1 (Pedrotti 11-12)

(a) eq(11-40)

$$M_{\max} = \frac{2t}{\lambda/n} = \frac{2nd}{\lambda} = \frac{2 \cdot 4.5 \cdot 2\text{cm}}{5.46 \times 10^{-7}\text{cm}}$$

$$M_{\max} = 329670$$



(b) eq(11-30)

$$F \equiv \frac{4r^2}{(1-r^2)^2} = 360$$

$$\Rightarrow \frac{T_{\max}}{T_{\min}} = 361$$

eq(11-32)

$$F = \frac{T_{\max} - T_{\min}}{T_{\min}} = \frac{T_{\max}}{T_{\min}} - 1$$

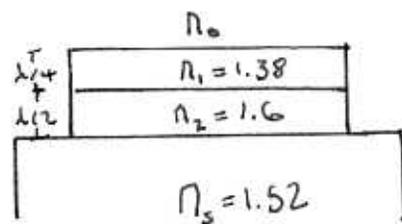
(c)

$$R = \frac{\pi}{2} m \sqrt{F} = \frac{\pi}{2} (329670) \sqrt{361} = 9.83 \times 10^6 = R$$

#2 (Pedrotti 19-9)

eq.(19-24)

$$M = \begin{bmatrix} \cos \delta & i \sin \delta / \gamma_1 \\ i \gamma_1 \sin \delta & \cos \delta \end{bmatrix}$$



$$\lambda/4 \rightarrow M_{\delta=\pi/2} = \begin{bmatrix} 0 & i/\gamma_1 \\ i\gamma_1 & 0 \end{bmatrix}$$

$$\lambda/2 \rightarrow M_{\delta=\pi} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$M_{\text{total}} = M_{\lambda/2} M_{\lambda/4} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & i/\gamma_1 \\ i\gamma_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i/\gamma_1 \\ -i\gamma_1 & 0 \end{bmatrix}$$

eq (19-36) for M_{total}

$$\Gamma = \frac{\gamma_0 M_{11} + \gamma_0 \gamma_3 M_{12} - M_{21} - \gamma_3 M_{22}}{\gamma_0 M_{11} + \gamma_0 \gamma_3 M_{12} + M_{21} + \gamma_3 M_{22}} = \frac{-i\gamma_0 \gamma_3 / \gamma_1 + i\gamma_1}{-i\gamma_0 \gamma_3 / \gamma_1 - i\gamma_1}$$

$$R_{\text{total}} = \Gamma^2 = \left(\frac{\gamma_0 \gamma_3 / \gamma_1 - \gamma_1}{\gamma_0 \gamma_3 / \gamma_1 + \gamma_1} \right)^2$$

for $M_{\lambda/4}$

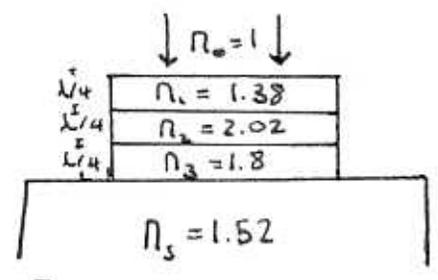
$$\Gamma = \frac{i\gamma_0 \gamma_3 / \gamma_1 - i\gamma_1}{i\gamma_0 \gamma_3 / \gamma_1 + i\gamma_1}$$

$$R_{\lambda/4} = \Gamma^2 = \left(\frac{\gamma_0 \gamma_3 / \gamma_1 - \gamma_1}{\gamma_0 \gamma_3 / \gamma_1 + \gamma_1} \right)^2$$

Thus,

$$\boxed{R_{\text{total}} = R_{\lambda/4}}$$

#3 (Pedrotti 19-11)



$$M_{total} = M_1 M_2 M_3$$

$$= \begin{bmatrix} 0 & i/\gamma_1 \\ i\gamma_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & i/\gamma_2 \\ i\gamma_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & i/\gamma_3 \\ i\gamma_3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & i\gamma_1 \\ i\gamma_1 & 0 \end{bmatrix} \begin{bmatrix} -\gamma_2\gamma_3 & 0 \\ 0 & -\gamma_2/\gamma_3 \end{bmatrix} = \begin{bmatrix} 0 & -i\gamma_2/\gamma_1\gamma_3 \\ -i\gamma_1\gamma_3/\gamma_2 & 0 \end{bmatrix}$$

From eq (19-36)

$$r = \frac{\gamma_0 M_{11} + \gamma_0 \gamma_5 M_{12} - M_{21} - \gamma_5 M_{22}}{\gamma_0 M_{11} + \gamma_0 \gamma_5 M_{12} + M_{21} + \gamma_5 M_{22}}$$

$$= \frac{\gamma_0 \gamma_5 (-i\gamma_2/\gamma_1\gamma_3) + i\gamma_1\gamma_3/\gamma_2}{\gamma_0 \gamma_5 (-i\gamma_2/\gamma_1\gamma_3) - i\gamma_1\gamma_3/\gamma_2}$$

$$= \frac{-\gamma_0 \gamma_5 \gamma_2^2 + \gamma_1^2 \gamma_3^2}{-\gamma_0 \gamma_5 \gamma_2^2 - \gamma_1^2 \gamma_3^2}$$

eq (19-12) $\gamma_i = n_i \sqrt{\epsilon_0 \mu_0} \cos \theta_{ti}$
 light normal so $\theta_{ti} = 0 \Rightarrow \gamma_i = n_i \sqrt{\epsilon_0 \mu_0}$

$$= \frac{-\sqrt{\epsilon_0 \mu_0} n_0 n_5 \sqrt{\epsilon_0 \mu_0} n_2^2 \epsilon_0 \mu_0 + n_1^2 \epsilon_0 \mu_0 n_3^2 \epsilon_0 \mu_0}{-n_0 \sqrt{\epsilon_0 \mu_0} n_5 \sqrt{\epsilon_0 \mu_0} n_2^2 \epsilon_0 \mu_0 - n_1^2 \epsilon_0 \mu_0 n_3^2 \epsilon_0 \mu_0}$$

$$= \frac{-n_0 n_5 n_2^2 + n_1^2 n_3^2}{-n_0 n_5 n_2^2 - n_1^2 n_3^2}$$

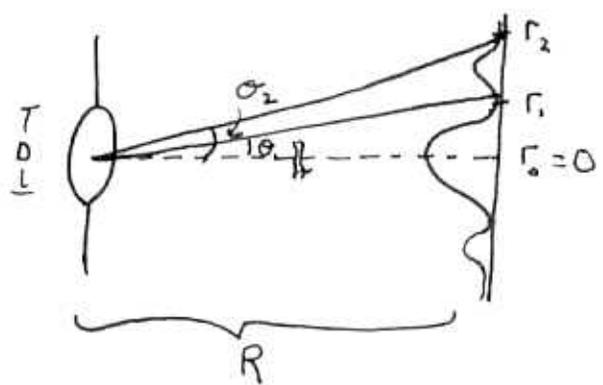
We want zero reflectance $r=0$

$$\Rightarrow n_1^2 n_3^2 = n_0 n_5 n_2^2 \Rightarrow \left(\frac{n_1 n_3}{n_2} \right)^2 = n_0 n_5$$

$$\Rightarrow \boxed{\frac{n_1 n_3}{n_2} = \sqrt{n_0 n_5}}$$

#4 (Pedrotti 16-7)

$D = 36 \text{ in} = .9914 \text{ m}$
 $f = 56 \text{ Ft} = \text{Distance from the aperture to screen}$
 $= R = 17.07 \text{ m}$



$\gamma = \frac{kD}{2} \sin \theta \Rightarrow$
 eq(16-20)

$\theta_1 = \frac{\gamma_1}{\pi} \frac{\lambda}{D} = \frac{5.14}{\pi} \frac{5.5 \times 10^{-7} \text{ m}}{.9914 \text{ m}}$

$r_1 = R \theta_1 = 1.55 \times 10^{-5} \text{ m}$

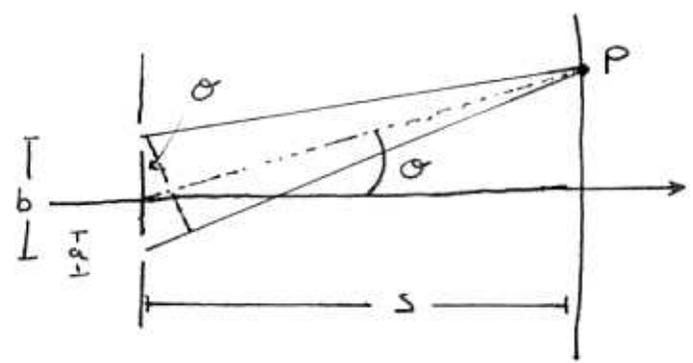
$r_2 = R \theta_2 = R \frac{\gamma_2}{\pi} \frac{\lambda}{D} = 17.07 \cdot \frac{8.42}{\pi} \cdot \frac{5.5 \times 10^{-7} \text{ m}}{.9914 \text{ m}}$

$r_2 = 2.54 \times 10^{-5} \text{ m}$

#5 (Pedrotti 16-13)

(a) Double Slit Diffraction Pattern:

$\lambda = 5.461 \times 10^{-7} \text{ m}$
 $b = 0.100 \text{ mm} = 1.00 \times 10^{-4}$



Assuming that the fourth-order maximum is the first one missing from the pattern, which implies that this missing order is due to $m=1$ diffraction minimum:

eq(16-30) $a = \left(\frac{p}{m}\right) b = \frac{4}{1} \cdot 1.00 \times 10^{-4} \text{ m} = 4.00 \times 10^{-4} \text{ m} = \text{slit separation}$

(b) From eq. (16-27),

$$I = 4I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha ; \quad \alpha = \frac{1}{2} k a \sin \theta$$

$$\beta = \frac{1}{2} k b \sin \theta$$

• For the zeroth order max. $\theta \rightarrow 0 : \cos^2 \alpha \rightarrow 1$

and $\frac{\sin \beta}{\beta} \rightarrow \cos \beta \rightarrow 1$ (L'Hopital's Rule)

So, $I_{0\max} = 4I_0$

• I_1 occurs at $p=1$

eq. (16-29) $p\lambda = a \sin \theta$

$$\Rightarrow \sin \theta = \lambda/a \Rightarrow \alpha = \frac{1}{2} \frac{2\pi}{\lambda} a \frac{\lambda}{a} = \pi$$

$$\beta = \frac{1}{2} \frac{2\pi}{\lambda} b \cdot \lambda/a = \pi \frac{b}{a} = \frac{\pi}{4}$$

$$\Rightarrow I_1 = 4I_0 \left(\frac{\sin \pi/4}{\pi/4} \right)^2 (-1)^2 = \frac{32}{\pi^2} = I_0$$

$$\Rightarrow \frac{I_1}{I_0} = \frac{8}{\pi^2} = .811$$

• I_2 at $p=2 \Rightarrow \sin \theta = \frac{2\lambda}{a} \Rightarrow \frac{I_2}{I_0} = \left(\frac{\sin \pi/2}{\pi/2} \right)^2 \Rightarrow \frac{I_2}{I_0} = \frac{4}{\pi^2} = .405$

• I_3 at $p=3 \Rightarrow \sin \theta = \frac{3\lambda}{a} \Rightarrow \frac{I_3}{I_0} = \left(\frac{\sin 3\pi/4}{3\pi/4} \right)^2$

$$\Rightarrow \frac{I_3}{I_0} = \frac{8}{9} \frac{1}{\pi^2} = .090$$

#6. (Pedrotti 16-22)

$$J_1(x) = \frac{\sin x - x \cos x}{\sqrt{\pi x}} \quad \text{for large } x$$

$$\Rightarrow J_1(x) = 0 \quad \text{when } \sin x = x \cos x$$

$$\Rightarrow \text{Or, } x_n = \frac{(2n+1)\pi}{4} \quad n=0, 1, 2, \dots$$

Now, from eq. (16-20)

$$\textcircled{1} \gamma_n = \frac{kD}{2} \sin \theta_n = \left(\frac{2n+1}{4} \right) \pi$$

↑
angle of diffraction min

$$\textcircled{2} \gamma_{n+1} = \frac{kD}{2} \sin \theta_{n+1} = \left(\frac{2(n+1)+1}{4} \right) \pi \quad \Delta\sigma = \theta_{n+1} - \theta_n \ll 1$$

$$\Rightarrow \sin \theta_{n+1} = \sin(\theta_n + \Delta\sigma) \approx \sin \theta_n + \Delta\sigma \cos \theta_n$$

Plugging in eq ① and ② into the above,

$$\Rightarrow \frac{2(n+1)+1}{4} \pi = \frac{2n+1}{4} \pi + \Delta\sigma \cos \theta_n \frac{kD}{2}$$

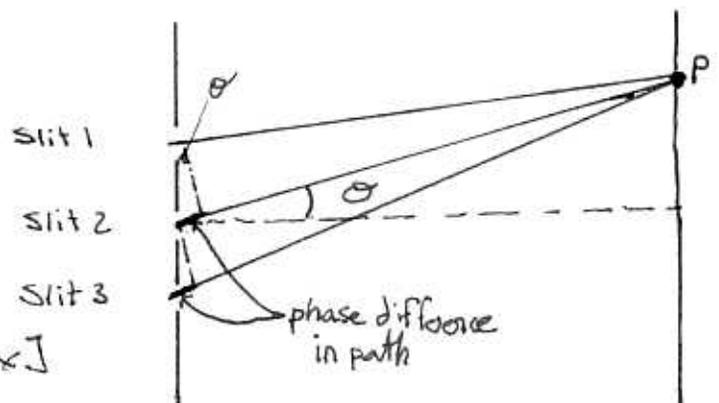
$$\Rightarrow \frac{\pi}{2} = \frac{kD}{2} \Delta\sigma \cos \theta_n \Rightarrow \boxed{\Delta\sigma = \frac{\lambda}{D \cos \theta}}$$

#7 (Pedrotti 16-25)

(a) $I_p = E_p^2 = 0$

$$\Rightarrow E_p = \tilde{E}_1 + \tilde{E}_2 + \tilde{E}_3$$

[The tildos '~' indicated the fields are complex.]



The middle slit creates a field equal to

$$\tilde{E}_2 = E_0$$

The other two slits will be equal amounts out of phase w/ the middle slit. E_1 will be ahead by φ , while E_3 will lag by φ_2 :

$$\tilde{E}_1 = E_0 e^{i\varphi} \quad \text{and} \quad \tilde{E}_3 = E_0 e^{-i\varphi}$$

So,

$$E_p = E_0 (1 + e^{i\varphi} + e^{-i\varphi}) = 0$$

$$\Rightarrow 2 \cos \varphi = -1 \Rightarrow \boxed{\varphi = \frac{4\pi}{6} = 120^\circ}$$

$$(b) \quad \varphi = \pi \Rightarrow E_p = E_0 (e^{i\pi} + 1 + e^{-i\pi}) = -E_0$$

$$\Rightarrow I_p = E_p^2 = E_0^2$$

$$\Rightarrow \boxed{\frac{I_p}{I_0} = \frac{1}{9}}$$

$$\left[\begin{array}{l} \text{see page} \\ 344 \end{array} \right] I_{\text{max}} = N^2 I_0 = 9 I_0$$

↑
number of slits

(c) The first principal maximum is the central max ($p=0$)

$$\Rightarrow \boxed{I_p = I_{\text{max}}}$$

(d)

① Suppose we had light incident upon the three slits such that the light coming from them was mutually incoherent. In this case the irradiance pattern would be uniform.

② Now, if we make them coherent, then the maximum irradiance goes up by a factor of 3, but the average stays the same.

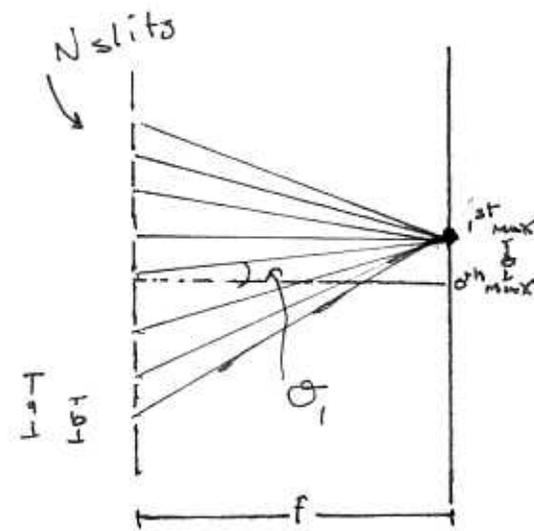
$$\text{i.e.} \quad \boxed{\frac{I_{\text{max}}}{I_{\text{av}}} = 3}$$

#8 (Pedrotti 17-8)

(1) $N=2$ (2) $N=10$ (3) $N=15,000$

$f = 2\text{m}; a = 5 \times 10^{-6}\text{m}; b = 1 \times 10^{-6}\text{m}$

$\lambda = 5.46 \times 10^{-7}\text{m}$

(a) Separation between 0^{th} & 1^{st} order max:

- ① This is dependent on the separation between slits,
- ② but not the number of slits N .

Thus, the result is the same in all three cases:

eq(16-34) $m\lambda = a \sin \theta \Rightarrow m=0 \rightarrow \theta_0 = 0$

$m=1 \rightarrow \theta_1 = \sin^{-1} \frac{\lambda}{a} = \sin^{-1} \left(\frac{5.46 \times 10^{-7}\text{m}}{5 \times 10^{-6}\text{m}} \right)$

$$\Delta y = f \theta_1 = (2\text{m}) \cdot 109 = 21.8\text{cm} \quad \theta_1 = .109$$

(b) We have eq.(16-30)

condition for missing order $a = \left(\frac{p}{m} \right) b$

central diffraction envelop $m=1$

interference max. integer

diffraction min integer

thus, $p_{\text{max}} = \frac{a}{b} = \frac{.005\text{mm}}{.001\text{mm}} = 5 \Rightarrow$ So, there are **Nine** bright fringes, for each number of slits $p = (0, \pm 1, \pm 2, \pm 3, \pm 4)$.

The last two $p_{\text{max}} = \pm 5$, are not peaks

(c) The central fringe is centered at $\theta=0$ and ends at the first interference minimum, which is at

eq(16-32) $I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2 = 0 \Rightarrow N\alpha = \pi$

$$\Rightarrow N \frac{ka}{2} \sin \sigma = \pi$$

$$\sin \sigma = \frac{2\lambda}{Na}$$

$$\sigma_N = \sin^{-1} \left(\frac{2\lambda}{Na} \right)$$

Therefore,

$$W_n = f 2\sigma_N \Rightarrow$$

$$W_2 = 21.8 \text{ cm}$$

$$W_{10} = 4.37 \text{ cm}$$

$$W_{10000} = 2.9 \times 10^{-5} \text{ m}$$